Einstein's 'Zurich Notebook' and the Genesis of General Relativity

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Abstract

Einstein saw his work on general relativity as something quite unique in his life. He felt that, if he had not created the special theory of relativity, someone else would have done so. His approach to a new theory of gravitation was entirely his own, carried through with considerable hard work and facing the skepticism if not active hostility from physicists he respected, such us Max Planck and Max Abraham. He characterized his efforts on special relativity as mere child's play compared to what was needed to complete general relativity.

There is a very important interpretative tool for understanding Einstein's search for the gravitational field equation known as the Einstein's "Zurich Notebook", a document written between 1912 and 1913. A group of scholars, John Stachel, John D. Norton and Jürgen Renn among others, have shown that the clarification reached by deciphering these Einstein's research notes has serious consequences for our understanding the genesis of general relativity. According to them, the Zurich Notebook shows that in 1912–1913 Einstein had already come within a hair's breadth of the final general theory of relativity. He failed, however, to recognize then the physical meaning of his mathematical results. Finally, in November 1915 he achieved the completion of the general theory of relativity. These recent discoveries are here remembered, in this Conference in memory of Aureliano Mira Fernandes and his time.

1 The long journey from Special to General Relativity

In 1905, Einstein created the Special Relativity (SR) – what he called then the Principle of Relativity – to reconcile the relativity of motion of inertial observers with electromagnetic theory of James Clerk Maxwell (1831–1879). In November of 1915, Einstein came out with General Relativity (GR) to reconcile gravity with the principles of SR and to extend the relativity

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of motion to include all observers. These are the main pointers we are here remembering.

Einstein himself, when he was preparing some notes for Erwin Freundlich's book, the first popular book on general relativity, published in 1916, divided the sequence of events of his search in three parts:

(a) In 1907, he had the basic idea for a generalized theory of relativity when he found a fundamental explanation for the equality of gravitational and inertial mass.

(b) In 1912, he finally recognizes the non-Euclidean nature of the space-time metric, and its physical determination by gravitation.

(c) In 1915, he arrives at the correct field equations for gravitation, and the explanation of the anomalous precession of the perihelion of Mercury.

Let us start in September 1907, when Einstein had agreed to write a review article, on his 1905 special theory of relativity, which was commissioned by Johannes Stark for the Jahrbuch für Radioaktivität und Electronik (Yearbook of Racdioactivity and Electronics). Einstein had only two months to write his Jahrbuch article. In this report, untitled "On the Relativity Principle and the Conclusions Drawn from it", Einstein gave an excellent overview of the principle of relativity for electrodynamics, mechanics, and thermodynamics. In a letter from November 1, 1907, Einstein informed J. Stark that he had "so arranged the work that anyone could find his way with comparative ease into relativity theory and its applications so far." When the article was completed on December 1, 1907, the first four parts devoted to a overview of the foundations of the principle of relativity, were followed by a fifth part, on nine pages, containing entirely new material. This last section, under the heading "The Relativity Principle and Gravitation," was committed to gravity. In it, Einstein argued that a satisfactory theory of gravity cannot be achieved within the framework of special relativity and that a generalization of that theory was needed. Namely, once the special theory was limited to "inertial systems," that is, to reference frames in uniform nonaccelerated motion relative to one another, Einstein raised the question of whether the principle of relativity could be extended to accelerated motion. This question could be understood as asking whether the covariant group of special theory of relativity could be extended beyond the Lorentz group. At the time Einstein did not feel prepared to deal with that question. Eventually, he made a connection between the generalization to any system of reference on one hand and the relativistic treatment of gravity on the other. This was a great and lonely step forward, since other researchers, such as Henri Poincaré, Hermann Minkowski, or Gustav Nordstrom, to mention just a few, felt that a perfectly adequate theory of gravitation could be constructed simply by modifying Newton's theory of gravitation somewhat to meet the demands of special relativity. Einstein, on the contrary, was looking for a generalization of the principle of relativity while was working on "Relativity Principle and Gravitation." Suddenly he found his way out when he was looking for a fundamental explanation for the equality of inertial and gravitational mass. Then a thought occurred to him that he later described as "the happiest thought of my life" [14].

Starting from a "thought experiment", of a man falling freely from the roof of a house, Einstein realized that he would be able to treat gravitation within the framework of special relativity. He realized that for such an observer "there exists – at least in its immediate surroundings – no gravitational field." Everything happens as if he were at rest or in a state of uniform motion. In the last part of his 1907 article, Einstein deals with the "relativity principle and gravitation", where he tackles the problem of generalizing the principle of special relativity to accelerated frames. Using the principle of equivalence, which also states that there is no difference between a uniform gravitational field and a uniformly accelerated frame, it is possible to go from one to another. This approach prove resourceful and provided Einstein with a plan to fight for the next eight years in the battle that would lead him to produce a theory of gravitation compatible with SR.

Then, in 1911, shortly after his arrival in Prague, where he became full Professor at the local German University, Einstein published a paper in the Annalen der Physik untitled "On the Influence of Gravity on the Propagation of Light." As he explained at the beginning of the paper, he was regressing to a subject of 1907. Indeed, his problems were still the same as in 1907, and his methods were almost the same, and for that reason obtained identical formulas. But some arguments, such as that of the red shift were new. However, he had come to realize that one of the most important consequences of that analysis was accessible to experimental test. In his own words, 'according to the theory I am going to set forth, rays passing near the sun experience a deflection by its gravitational field, so that a fixed star appearing near the sun displays an apparent increase of its angular distance from the latter, which amounts to almost one second of arc." [3].

That is, Einstein's search for general relativity spanned an eight years interval, 1907–1915. But it is fair to say that some periods were calm and some were more forceful. The moment when the great development comes about was sometime between the late summer of 1912, when Einstein moved from Prague to Zurich, and early 1913. Prior to his move to Zurich in August 1912, Einstein was already struggling with the puzzle of accommodating gravitation into his 1905 special theory of relativity. Then he saw it, the connection between gravity and non-Euclidean geometry. One could introduce the most general gravitational fields in the space-time of special relativity merely by curving its geometry. All this thought and knowledge travels with him when he goes back to his Alma Mater, the Zurich Polytechnic.

Summarizing, Einstein's approach was embodied in heuristic principles that guided his search from the beginning in 1907. The first and more lasting one was the 'Equivalence Principle" which states that gravitation and inertia are essentially the same. This insight implies that the class of global inertial frames singled out in the special relativity can have no place in a relativistic theory of gravitation. In other words, Einstein was led to generalize the principle of relativity by requiring that the covariance group of his new theory of gravitation be larger than the Lorentz group. This will lead him trough a long journey and in his first step, already in his review of 1907, Einstein formulated the assumption of complete physical equivalence between a uniformly accelerated reference frame and a constant homogeneous gravitational field. That is, the principle of equivalence extends the covariance of special relativity beyond Lorentz covariance but not as far as general covariance. Only later Einstein formulates a "Generalized Principle of Relativity" which would be satisfied if the field equation of the new theory could be shown to possess general covariance. But Einstein's story appealing to this mathematical property, general covariance, is full of ups and downs.

The turning point in the history of Einstein's discovery of the gravitational field equations is in the early summer of 1912, when he realizes the significance of the metric tensor and the general line element for a generalized theory of gravitation ([27, §12b], and [31]). Then Einstein started to study properly the mathematics of Gaussian surface theory, apparently in collaboration with Grossmann,[2] becoming acquainted with Beltrami invariants. Grossman also discovered for Einstein the existence of the "absolute differential calculus" of Ricci and Levi-Civita [29] that would enable Einstein to construct a generally covariant theory of gravitation. However, when Einstein and Grossmann published the results of their own research in early 1913, the theory of the resulting paper, commonly known as the "Entwurf" theory from the title of

[2]The lectures of Professor Carl Friederich Geiser, which Einstein heard as a student at the ETH, had familiarized him with Gaussian theory of two-dimensional surfaces.

the paper [17], failed to comply with the generalized principle of relativity, since this theory offered a set of gravitational field equations that was not generally covariant.

Until the fall of 1915 Einstein continued to elaborate on and improve the "Entwurf" theory and explored many of its consequences. Already in 1913, Einstein and his friend Michele Besso had found that "Entwurf" equations did not account for the anomalous advance of the perihelion of Mercury, something that Einstein hoped to explain with his new theory of gravitation.[3] Although Einstein knew the failure of the "Entwurf" theory to resolve the Mercury anomaly he continues to hold on to this theory in spite of everything. How he had overcome all obstacles and finally obtained in November 25, 1915 his final theory of gravitation will be explained in what follows.

In the "Autobiographical Notes," Einstein points out that the importance of the equivalence principle in requiring a generalization of SR was clear to him in 1908 (actually it was in 1907). And he adds: "Why were another seven years required for the construction of the general theory of relativity? The main reason lies in the fact that it is not so easy to free oneself from the idea that co-ordinates must have an immediate physical meaning." [15, p. 67]. What I said so far is not enough for you to understand this comment made by Einstein in 1949. But it was exactly the resolution of this puzzle that separates Einstein from the final theory, as you will discover if you read the whole paper.

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The task of a reconstruction of Einstein's building of the theory of general relativity has challenged several historians of science for a long time.[4] A major step forward in this venture is due to John Stachel's and John Norton's ground-breaking investigations.[5] A very important interpretative

[3]See Einstein to Conrad Habicht, December 24, 1907 [21, p. 82]: "At moment I am working on the relativistic analysis of the law of gravitation by means of which I hope to explain the still unexplained secular changes in the perihelion of Mercury."

[4]A very incomplete list of the older secondary literature certainly involves [23], [25], [1], and [27].

[5]See [31], [32], [33], and [26].

tool for understanding Einstein's search for the gravitational field equations is the so-called Einstein's Zurich Notebook,[6] a document written between summer 1912 and spring 1913, during his time in Zurich. It was in the course of preparing the editorial project of the Collected Papers of Albert Einstein that John Stachel first realized the importance of this manuscript for the reconstruction of the Einstein theory of gravitation [31]. A little later, in 1984, John Norton also published a comprehensive reconstruction of Einstein's discovery process [26]. Following these discoveries, a group of scholars including John Stachel, John D. Norton, Jürgen Renn, among others, undertook a systematic analysis of this notebook and revealed an unexpected result: Albert Einstein had written down, in 1912, an approximation to his final field equations of gravitation, which were later derived by him three years later. He failed, however, to recognize the physical meaning of his mathematical results. In 1997, Jürgen Renn and Tilman Sauer have shown that the clarification reached by deciphering Einstein's research notes would have serious consequences for our understanding of the genesis of general relativity [28]. According to them, the Zurich Notebook shows that in 1912–1913 "Einstein had already come within a hair's breadth of the final general theory of relativity." In any case, the period between 1913 and November 1915 should not be considered a period of stagnation. It was rather a period during which Einstein arrived at a number of insights that created the prerequisites for his final triumph

In what follows, I will recall the research carried out by these scholars deciphering Einstein's notebook, and the interplay between physics and mathematics, ignited by general relativity during Mira Fernandes lifetime.

2 Einstein's search for General Relativity in the Zurich Notebook

The Zurich Notebook originally comprised 96 pages. The notebook has two front covers. Einstein wrote in it in both directions. In the first front cover, Helen Dukas, Einstein's secretary's, typed description of the notebook as "Notes for Lectures on Relativity…" If we flip the notebook over, we find a second cover with the word "Relativität" in Einstein's hand writing. 84 pages of this notebook contain calculations or short notes on various problems of physics, mainly on gravitation theory. According to Jürgen Renn and Tilman Sauer [28], "most of calculations are extremely sketchy,

[6]Einstein Archives Call Nr. 3-006

display a lot of false starts, and come with no explanatory text." Inside the Relativity' cover are a few rough sketches and some recreational puzzles in mathematics. The page that faced it, however, contained serious physics. There we find Einstein recounting the elements of the four-dimensional approach to relativity and Minkowski's electrodynamics, starting with four space-time coordinates (x, y, z, ict)=(x^1, x^2, x^3, x^4) and going on through scalars, four-vectors and six-vectors and their operations. Let's recall that it took sometime for Einstein to embrace Minkowski's reformulation of special relativity in terms of a four-dimensional space-time manifold, which is a crucial instrument for the further development of a relativistic theory of gravitation. As late as July 1912, Einstein had not adopted the four-dimensional geometrical aproach of Minkowski, even though a book using this aproach had already been published [24]. Apparently, Einstein got acquainted with Minkowski formalism through Laue's book. All this changed with Einstein's move to Zurich in August 1912 where he began collaborating with his old mate Marcel Grossmann, now the chairman of the independent Section VIII of the Swiss Federal Technical University. Once there, Einstein was then introduced by Grossmann to the 'absolute differencial calculus' of Ricci and Levi-Civita.

But let's go back to Zurich Notebook pages where Einstein was starting to deal with Minkowski approach. A central element of Minkowski's geometrical representation of special relativity was the manifest invariance under linear, orthogonal transformations of the quantity

$$x^2 + y^2 + z^2 - (ct)^2 = (x^1)^2 + (x^2)^2 + (x^3)^2 - (x^4)^2\,,$$

or in differential form

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2\,, \tag{1}$$

The development continues for 13 pages recounting notions in electrodynamics and thermal physics. All of a sudden, without a word of warning of the new subject, we stumble on the basic notion of general relativity, the "line element" written at the top of the page 39L, the first exploration of a metric theory. In that page, Einstein gathered the building blocks for his new theory of gravitation: the metric tensor, and the gravitational equation of his second theory of static gravitational fields. This was quite possibly the first time Einstein has written down this expression. The coefficients $G_{\mu\nu}$ of what we now know as the 'metric tensor" are written with an upper case G. Einstein changed within a few pages to the lower case g, which remained his standard notation from then on,

$$ds^2 = \sum_{\alpha,\beta=1}^{4} g_{\alpha\beta} dx^\alpha dx^\beta \, . \tag{2}$$

For Einstein at that time, the big project was to find how this quantity $g_{\mu\nu}$, the metric tensor, is generated by sources (masses or fields). Eventually, this would lead to the new gravitational field equation, that is, Einstein's theory analog of Newton's inverse square law of gravity. The lower half of the page is clearly making rudimentary efforts in that direction. There Einstein chooses a "Spezialfall" – a special case – in which the coefficients of the metric tensor revert to the values of special relativity, excepting $G_{44} = -c^2$. The coefficients $G_{\mu\nu}$ enable us to compute the spatio-temporal interval ds^2 between events separated by infinitesimal coordinate differences dx^μ. If these coefficients assign spatio-temporal intervals that do not conform to a flat geometry, then we have captured the full range of gravitational effects in the manner of Einstein's general theory.

Before we go any further in explaining the content of the notebook, let us recall the principal steps taken by Einstein in his path towards a new theory of gravitation. Then we will be able to establish the connection between those steps and the notebook's subject matter. As pointed out above, in 1907 when Einstein was still at the patent office in Bern he had already discovered a practical way to deal with gravity and with accelerated observers. He realized then that the effects of acceleration were indistinguishable from the effects of gravity. Somehow, Einstein succeeded in unifying all kinds of motion. Uniform motion is indistinguishable from rest, and acceleration is no different from being at rest in a gravitational field, at least locally. The crucial elements for that purpose were the Principle of Equivalence and the Generalized Principle of Relativity. Einstein saw a Generalized Principle of Relativity as guaranteeing the satisfaction of the Equivalence Principle (EP). Indeed, according to the EP, an arbitrarily accelerated frame of reference in Minkowski space-time can precisely then be considered as being physically equivalent to an inertial frame provided a gravitational field is introduced which accounts for the inertial effects in the accelerated frame. As early as 1907 he had come to consider two possible physical consequences of the EP, the bending of light in a gravitational field and the gravitational red-shit.

Resuming our description of Zurich Notebook where we left, Einstein then employs the equivalence principle to interpret the matrix elements of eq. (2), $g_{\mu\nu}$, that had arisen with the introduction of arbitrary coordinates. In the special case of the EP, the transformation from (1) to (2) is from an inertial coordinate system to a uniformly accelerated coordinate system, but where c now is a function of the spatial coordinates (x', y', z'). That is, (1) is now

$$ds^2 = -c^2(x', y', z')dt'^2 + dx'^2 + dy'^2 + dz'^2\,, \tag{3}$$

According to the EP, the presence of a gravitational field is the only difference between the space-time (3) and that of special relativity (1). Consequently, based on the EP, Einstein was led to interpret the line element (3) as representing a static gravitational field with the coordinate dependent $c(x', y', z')$ of (3) representing the gravitational potential, and the $g_{\mu\nu}$ of (2) as representing a more general gravitational field.

The differential equation for the velocity of light c written on the bottom of page 39L may be recognized as the left hand side of a nonlinear generalization of the classical scalar Poisson equation for the field c representing the potential of the static gravitational field

$$c\Delta c - \frac{1}{2}(\mathrm{grad}\, c)^2 = \kappa c^2 \sigma\,, \tag{4}$$

which Einstein had published in March 1912 [5]. On the right hand side of equation (4) κ e is a constant, and σ is denoting the field generating mass and (non-gravitational) energy density.

To set the differential equation (4) into a form which allowed its interpretation as one particular component of a 10-component tensorial field equation for the metric tensor, which is a second order symmetric tensor, Einstein performs the transformation $c^2/2 = \gamma$. The transformed equation would thus represent a special limiting case, for static fields, of the general tensorial field equation. For Renn and Sauer [28] this is a good example of the central question concerning all calculations of the Zurich Notebook coping with the problem of gravitation: *What is the appropriate differential expression* $\Gamma_{\mu\nu}$ *which is formed from the metric tensor and its first and second derivatives and which enter a field equation of the form*

$$\Gamma_{\mu\nu} = \kappa T_{\mu\nu}\,, \tag{5}$$

with the stress-energy-tensor $T_{\mu\nu}$ of *matter (and energy) as the source term on the right hand side?*

However, at this point it is very obvious that Einstein has not yet used any of the techniques of the Ricci and Levi-Civita absolute differential calculus, now called "tensor calculus." Instead he uses older methods due to Beltrami to see what invariant quantities can be formed from a scalar φ. He starts with simple questions. As a first attempt, he looks at the coordinate divergence of the metric tensor and asks, "Ist dies invariant?" — "Is this invariant?" As the calculation that follows immediately shows in the notebook, it is not. How these quantities transform under change of coordinates is clearly a major focus of his analysis. The analysis continues for three more pages and then we find one of the most fascinating pages.

Here Einstein finds several of the key notions of his new theory in a familiar ground: classical physics. Einstein re-derives a standard result: if a mass is free to move inertially, except that it is constrained to move within a curved surface, what is the curve traced by the mass on surface? It proves to be a geodesic of the surface, a curve of shortest distance. The result is very close to the central idea of Einstein's general theory of relativity, the final theory he obtains in 1915. The following table shows how close it comes.

Table 1: Classical Physics versus General Relativity

Classical physics	**General relativity**
A mass moves freely in space, except that it is constrained to a 2-dimensional surface in the 3-dimensional space.	A mass moves freely in space-time. That is, it is in free fall, so that gravity acts on it through the curvature of space-time.
Its spatial trajectory is a geodesic of the two-dimensional surface. That is, it traces of a curve of shortest length in the surface.	Its space-time trajectory is a geodesic of the space-time. That is, it traces a curve of extremal space-time interval in space-time.

The surface is defined by a scalar field f in space. Constant values of f, such as $f = 0$, pick out the surface. The equation of motion of the mass moving in the surface is just that its acceleration vector $(d^2x/dt^2, d^2y/dt^2, d^2z/dt^2)$ is proportional to the reaction force from the surface, which is orthogonal to the surface and proportional to the gradient of f $(\partial f/\partial x, \partial f/\partial y, \partial f/\partial z)$ All this is summarized in:

$$\begin{aligned} m\frac{d^2x}{dt^2} &= \frac{\lambda}{m}\frac{\partial f}{\partial x} = \lambda' \frac{\partial f}{\partial x},\ f = 0. \\ --- &= --- \end{aligned}$$

Immediately below is a straightforward derivation where Einstein uses the variational principle: If a mass point in the surface obeys these equations of motion, then the spatial length of the path traced on the surface, $\int ds$, is extremal, in that it satisfies the condition $\delta \int ds = 0$. These computations proceed until we reach a page on which calculations from each side of the notebook meet. Flipping the notebook over and starting the other side one can see a series of pages of computation in the statistical-thermal physics of heat radiation. After nine pages like this, Einstein starts a new heading, "Gravitation," and we are finally deep into a considerable discussion of general relativity.

On this page Einstein sets up the equations for conservation of energy and momentum for continuous matter in general relativity. He starts with the equation of motion for a point mass–the geodesic equation–but now written in the form of an Euler-Lagrange equation:

$$\frac{d}{dt}\frac{\partial H}{\partial \dot{x}} - \frac{\partial H}{\partial x} = 0, \quad \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = -\frac{\partial \Phi}{\partial x}$$

He then applies this to a cloud of non-interacting dust particles in free fall to arrive at what we now recognize to be the condition of the vanishing of the covariant divergence of the symmetric stress-energy tensor $T_{\mu\nu}$,

$$\frac{\partial}{\partial x^{\nu}}\left(\sqrt{-g}g_{\mu\alpha}T^{\mu\nu}\right) - \frac{1}{2}\sqrt{-g}\frac{\partial g_{\mu\nu}}{\partial x^{\alpha}}T^{\mu\nu} = 0\,, \tag{6}$$

which in modern notation is given by $T^{\nu}_{\alpha;\nu} = 0$. Einstein arrives at eq. (6) by the fall of 1912, when he was looking for a generalization of the special relativistic formulation of the conservation of energy and momentum as well as of the Newtonian law of motion for continuous matter in a gravitational field. Renn and Sauer [28] call this requirement the "Conservation Principle", one of four heuristic requirements that Einstein took along to check each of the candidate field equations. The other three requirements were the equivalence principle, the generalized principle of relativity, and the requirement of correspondence. According to this last requirement, the new theory should describe, under certain limiting conditions, the gravitational effects familiar from Newtonian physics. In other words, Einstein expected that the unknown gravitational field equation for the metric tensor would reduce to the Poisson equation for the scalar gravitational potential of the classical theory. This explains the equation (4) above, and all his attempts to find the appropriate left hand side for equation (5). Then, under some limiting conditions, the equation of motion of his new theory would yield Newton's second law. Therefore, Einstein assumed that the Newtonian limit of his theory could be obtained from the full equation with the limiting condition of weak static fields leading to a linearized field equation for the metric tensor, that is via a metric of the form (3). However, just looking for the Notebook, there is good evidence that Einstein's knowledge of tensor calculus is still limited. He does not know or is not sure that the operator acting on $T_{\mu\nu}$ in this equation is a generally covariant operator. To check the operator, he replaces $T_{\mu\nu}$ by the tensor $g_{\mu\nu}$ and sees whether the result is zero or a four vector, as it should be if the operator is generally covariant. It proves to be zero and Einstein is pleased.

After this, the pages continue with increasingly elaborate attempts to form invariant quantities from the metric tensor, most probably with the intention of finding a generally covariant set of gravitational field equations. But still, at this point, the techniques of Ricci and Levi-Civita for producing invariant quantities from the derivatives of the metric tensor are absent. Most significantly, there is no sign of the fourth rank Riemann curvature tensor from which we now know the gravitational field equations are readily constructed.

3 Genesis of General Relativity: a drama in 3 acts

Following John Stachel, we may describe the genesis of general relativity as a drama in three acts:

i) First act (1907), Einstein adopts the Equivalence Principle (EP), i.e., all bodies fall with same acceleration in a gravitational field as a criterion for building the theory;

ii) Second act (1912), Einstein concludes that any scalar generalization of Newton's theory would not be adequate since it violates the Equivalence Principle. That is, Einstein adopts the EP as a chief criterion for the construction of his new theory of gravity. Then he assumes the need for a curved space-time, through the metric tensor:

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta \tag{7}$$

iii) Third act (1915): the covariance of the field equations – by 1913 he had convinced himself that generally covariant field equations are physically inadmissible since they cannot determine the metric field uniquely, and he has been fighting with this for a couple of years. Eventually, in November 1915 he arrives at the final covariant equations

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 8\pi G T_{\alpha\beta}\,. \tag{8}$$

So far we have considered in some detail the first two acts. Let us take a close look at the third one, the problem of covariance of the gravitational field equation. In the 1915 paper [11] as well as in the other papers of that period, including a paper from March 1918 where Einstein [13] returned to the question of the fundamental principles of general relativity, the equivalence principle is still understood as being included in the generalized principle of relativity which Einstein believed is satisfied because of the general covariance of the new field equation. Going back to 1912 and to Zurich Notebook we see, after the pages where Einstein is searching for invariants, he finally finds the first reference to Riemann tensor. Einstein writes at the head of the page the formula for the Riemann tensor, using the old "four-index symbol" notation, (ik, lm), with the following entry: "Grossmann tensor fourth rank." This clearly suggests that Grossmann passed on the formula to Einstein, proving the often-told story that Einstein only learned of the methods of Ricci and Levi-Civita through his school friend, the mathematician Marcel Grossmann. When he was searching of methods that could accommodate arbitrary coordinate systems, Grossmann found Ricci and Levi-Civita's article of 1901 with the Riemann tensor and reported it to Einstein. In October 1912, Einstein wrote to Arnold Sommerfeld: "I am now working exclusively on the gravitation problem and I believe that I can overcome all difficulties with the help of a mathematician friend of mine here." [21, p. 505]

In his attempt, with Marcel Grossmann, to find a gravitational field equation for the metric tensor, at the end of 1912 or in the beginning of 1913 Einstein came close to his final field equations when he considered the Ricci tensor, defined by the formula

$$R_{\alpha\beta} = \frac{\partial \Gamma^{\mu}_{\alpha\beta}}{\partial x^{\mu}} - \frac{\partial \Gamma^{\mu}_{\alpha\mu}}{\partial x^{\beta}} + \Gamma^{\mu}_{\alpha\beta}\Gamma^{\sigma}_{\mu\sigma} - \Gamma^{\lambda}_{\alpha\sigma}\Gamma^{\sigma}_{\beta\lambda} \tag{9}$$

where the objects,

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} \left(g_{\nu\alpha,\beta} + g_{\nu\beta,\alpha} - g_{\alpha\beta,\nu} \right)$$

represent the Christoffel symbols of second kind (these would be generalized, within a few years, to the concept of affine connection, by Levi-Cività, Weyl and Cartan).

In the notebook, Einstein proceeded in essentially the modern way. He contracted the fourth rank Riemann tensor to produce the symmetrical, second rank Ricci tensor. Before going any further Einstein inquired whether this new tensor would describe, under certain limiting conditions, the gravitational effects familiar from Newtonian physics. In other words, if it is to serve as a gravitation tensor it must reduce in the weak, static field to a Newtonian form. This requires three of its four second order derivative terms to vanish

$$\sum_k \left(\frac{\partial^2 g_{kk}}{\partial x^i \partial x^m} - \frac{\partial^2 g_{ik}}{\partial x^k \partial x^m} - \frac{\partial^2 g_{mk}}{\partial x^k \partial x^i} \right) = 0\,, \tag{10}$$

as Einstein commented in the notebook. Discovering how to eliminate these three terms in the weak field limit will become a major focus and major obstacle for Einstein in the pages to follow. Apparently this difficulty led Einstein to conclude that the Ricci tensor violated his requirement of correspondence, mentioned before. The usual interpretation is that Grossmann failed to see how to extract a d'Alembertian operator from the linearized approximation to the Ricci tensor because of his inability to choose a suitable coordinate condition. Indeed, in the second, "mathematical", part of Einstein-Grossmann paper which was written by Grossmann, after defining the Riemann tensor Grossmann explains how one can give a covariant differential tensor of second rank and second order, the Ricci tensor, and states: "But it turns out that this tensor does *not* reduce, in the special case of an infinitely weak static gravitational field, to the expression $\Delta\varphi$." [17, p. 257]

But according to [31], there is a better interpretation that depends on the existence of a physical misconception on the part of Einstein. Indeed, this misconception was responsible for his delay in finding a theory of gravitation capable of explaining the anomalous advance of Mercury, as we will see later. When he was working on the Einstein-Grossmann paper, Einstein took for granted that, even before having the correct field equations, he already knew the correct form of the metric tensor for a static gravitational field, based on his earlier work on the subject ([4], [5]). On page 229 of part I, he wrote the solution given by equation (3). It is easily shown that substituting this solution for the metric tensor into the formula for the Ricci tensor, neglecting all but the linear terms, because we are dealing with an "infinitely weak" field, it follows from $R_{\mu\nu} = 0$ that g_{44} can depend at most linearly from the coordinates. Thus, it cannot possibly represent the gravitational potential of any (finite) distribution of matter, static or otherwise. No wonder that Einstein gave up the Ricci tensor at this point.

In the meantime Grossmann have shown him how to derive a new candidate for the left hand-side of the gravitational field equation,

$$N_{\alpha\beta} = \frac{\partial \Gamma^{\mu}_{\alpha\beta}}{\partial x^{\mu}} - \Gamma^{\sigma}_{\alpha\mu}\Gamma^{\mu}_{\beta\sigma} \tag{11}$$

This object is easily obtained from the Ricci tensor. One may subtract from the Ricci tensor a part that transforms tensorially under the restricted group of unimodular coordinate transformations which leave $g = \det(g_{\mu\nu})$ invariant, that is,

$$\Gamma^{\mu}_{\alpha\mu\,;\beta} = \frac{\partial \Gamma^{\mu}_{\alpha\mu}}{\partial x^{\beta}} - \Gamma^{\sigma}_{\alpha\beta}\Gamma^{\mu}_{\sigma\mu} \tag{12}$$

and take the remaining part. It is quite clear that under this restricted group

$$\Gamma^{\rho}_{\alpha\rho} = \frac{\partial(\ln g)}{\partial x^{\alpha}}$$

transforms as a vector, and its covariant derivative transforms as a tensor. The "tensor" (11) was also reconsidered by Einstein in November 1915 [9], and for this reason was called the "November tensor" by Renn and Sauer [28]. The analysis of the notebook has revealed that at the end of 1912 or at the beginning of 1913 Einstein even happened to consider, in linearized form, the final equation of General Relativity, equation (8) above, where in its left hand side stands what is now called the Einstein tensor,

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\,, \tag{13}$$

and abandoned it as well, because they could not find the appropriate static Newtonian limit,[7] and they move on to other candidate field equation.

Instead of pursuing this covariant approach, the notebook ends with a derivation of the left hand side of the so-called "Entwurf" field equations,

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^{\alpha}}\left(\sqrt{-g}g^{\alpha\beta}\frac{\partial g_{\mu\nu}}{\partial x^{\beta}}\right) \quad - \quad g^{\alpha\beta}g^{\lambda\rho}\frac{\partial g_{\lambda\mu}}{\partial x^{\alpha}}\frac{\partial g_{\rho\nu}}{\partial x^{\beta}} \tag{14}$$

$$- \quad \frac{1}{2}\frac{\partial g_{\lambda\rho}}{\partial x^{\mu}}\frac{\partial g^{\lambda\rho}}{\partial x^{\nu}} + \frac{1}{4} g_{\mu\nu} g^{\alpha\beta}\frac{g_{\lambda\rho}}{\partial \alpha}\frac{g^{\lambda\rho}}{\partial \beta}\,. \tag{15}$$

This differential operator is covariant only under some restricted class of coordinate transformations, at least under linear transformations but the precise transformational properties were unknown to Einstein and Grossmann when they published their paper [17]. Giving up general covariance should not be an easy decision, mainly because Einstein was looking for generally covariant field equations from the start, and for him general covariance corresponds to the acceptability of arbitrary reference frames. In other words, this would constitute for Einstein an extension of the Principle of

[7]Carried out by Renn et al at the Max Planck Institute for the History of Science.

Relativity. On other hand, as I mention before, Einstein saw the Principle of Relativity as guaranteeing the satisfaction of the Equivalence Principle as well. Taking all this in consideration one should ask, how did Einstein find non-generally covariant gravitational field equations after all?

J. Stachel [31] explains that Einstein based his search on the criteria that the field equations should: (i) generalize Poisson's equation for the Newtonian gravitational potential; (ii) be invariant at least under linear transformations (as he put it – manifest no *less* relativity than in the special relativity); (iii) include a gravitational stress-energy complex, built from the metric tensor and its first derivatives (by analogy with Maxwell's electromagnetic theory), that is a tensor under linear transformations which enters the field equations as a source term in the same way as does the stress-energy tensor of ordinary (non-gravitational) matter or fields.

Based on these three requirements, Einstein was able to derive the set of gravitational field equations in part I of the Einstein-Grossmann paper. He also gave a sketchy proof of how the Newton's law of gravitation follows from the linear approximation of these equations for the static case. In particular, he stressed that the spatial metric remains flat in this approximation, as he was expecting. It seems that Einstein considered that the lack of general covariance could only be temporary. For example, in letter to Paul Ehrenfest on May 28, 1913 he wrote:

> I am now inwardly convinced that I have found that which is correct, and, at the same time, that a murmur of disappointment will, of course, go through the rank of our colleagues when the work appears, which will be in a few weeks ... The conviction to which I have slowly struggled through is that there are no preferred coordinate systems of any kind. However, I have only partially succeeded, even formally, in reaching this standpoint.

Conversely, by November 1913, Einstein had developed the "hole" argument against general covariance. He wrote to Ludwig Hopf on November 2:

> I am now very content with the gravitation theory. The fact that the gravitational equation are not generally covariant, which a short time ago still disturbed me so much, has proved to be unavoidable; it is easily proved that a theory with generally covariant equations cannot exist if one demands that the field be mathematically completely determined by matter.[8]

[8]Einstein Archives Call Nº 13-290.

The proof in question, alluded to in the letter, is the infamous 'hole" argument first published in the addendum to the "Entwurf" paper [17], signed by Einstein alone and not published in the original printing of the paper. Based on the letter to Hopf, quoted above, we can date the origin of the argument fairly closely. In his Vienna lecture of September 23, 1913, to the meeting of the Geselschaft Deutscher Naturforscher and Ärzte, he stated:

> In the last few days I have found the proof that such a generally covariant solutions cannot exist at all. [6, p. 1257, footnote 2].

Einstein repeats just about the same argument in two subsequent papers in 1914 and in several letters to friends and colleagues, and the core of his reasoning was complete by November 1913. In his clear formulation [8] the hole argument proceeds as follows. Let there be a region of space-time H (the "hole"), an open subspace of a manifold M, devoid of matter and energy and a set of generally covariant field equations valid for the entire space-time manifold M, both inside and outside H. Given a coordinate system of the manifold, K, then what happens physically in H is then completely determined by the solutions of the field equations, $g_{\mu\nu}$, the components of the metric tensor, as functions of the coordinates x_ν. The totality of these functions will be represented by $G(x)$. Consider now a second coordinate system, K', that coincides with K everywhere of and on the boundary of H, and within H it diverges from K but in such a way that the metric components $g'_{\mu\nu}$ referred to K', like the $g_{\mu\nu}$, and their derivatives, are everywhere continuous. The totality of the $g'_{\mu\nu}$ expressed in terms of the new coordinates x'_ν will also be represented by $G'(x')$. It is important to note, as Einstein did, that $G'(x')$ and $G(x)$ describe the same gravitational field. That is, they are two different mathematical representations of the same physical field. However, if we replace de coordinates x'_ν by the coordinates x_ν in the functions $g'_{\mu\nu}$ and represent them by $G'(x)$, then $G'(x)$ also describes a gravitational field with respect to K, which is different from the original gravitational field within the "hole", H. However the two different solutions $G'(x)$ and $G(x)$, which are written in the same coordinate system, correspond to the same "reality" (same sources and same boundary conditions). In summary, because generally covariant field equations admit non-equivalent solutions for events within H, such equations are not acceptable as an appropriate physical theory of gravitation. This is the "hole" argument against general covariance of the field equations. So, if we require that the course of events in the gravitational field be determined by the laws to be set up, we must therefore adopt a theory with restricted covariance properties. One could think that this Einstein's argument was a sort of excuse to accommodate his "Entwurf" theory with limited covariant properties. But, indeed, at the end of the day his argument was much deeper than that.

In trying to explain Einstein's line of reasoning behind his arguments, we kept as close as possible to the mathematical language and methods of his time. However, we must bear in mind that the modern terminology of differential geometry, which expresses geometrical concepts in coordinate-free language and distinguish between coordinate transformations and (active) diffeomorphisms was not available to Einstein. With modern terminology and methods one may more easily clarify these arguments.

Let's to reconsider the "hole" argument from a more modern perspective [30]: Assume that the gravitational field equations are generally covariant. Consider a solution of these equations in which the gravitational field is g and there is a region H of the universe without matter, the "hole". Assume that inside H there is a point A where g is flat and a point B where g is not flat. Consider a smooth map $\phi : M \to M$ which reduces to the identity outside H, and such that $\phi(A) = B$, and let $\tilde{g} = \phi^* g$ be the pull-back of g under ϕ. The two fields, g and $\tilde{g}$, have the same past, are both solutions of the field equations, but have different properties at the point A. Therefore, the field equations do not determine the physics at the space-time point A. That is, they are not deterministic. However, we know that (classical) gravitational physics is deterministic. So, one must pick one of the following: (i) the field equations must not be generally covariant, or (ii) there is no meaning in talking about the physical space-time point A. The correct physical conclusion is the second one, that there is no meaning in referring "the event A" without further specification.

By late 1915, after having returned to generally covariant field equations, Einstein introduced the point-coincidence argument, which maintains that a coordinatization of the manifold is itself not sufficient to determine an individuation of the points (events) of the manifold. Einstein now argues that the events of the space-time are implicitly defined and thus individuated only as points of intersection or coincidence of world lines. Then, as the coincidences are themselves determined by the metric, it is impossible to have two different sets of values of the functions $g_{\mu\nu}(x_\nu)$ assigned to *one and the same event* of the space-time manifold. Therefore, in regions where no matter is present, the points of a manifold are physically differentiated only by the properties that they inherit from the metric field. In general, a space-time manifold with metric field corresponds to a gravitational field; but a gravitational field corresponds to an equivalence class of manifolds with metric fields. In particular, any set of generally covariant field equations that has $G(x)$ as a solution in some empty region of space-time will also have $G'(x)$ as a solution in that region. $G(x)$ and $G'(x)$, together with all other mathematically distinct metric tensor fields that can be transformed into each other by being dragged along with an (active) diffeomorphism, form a equivalence class of solutions. But this equivalence class of mathematical distinct metric tensor fields corresponds to one physical solution to the field equations, that is, to one gravitational field. [34]

To clarify and give further support to the point-coincidence argument, Einstein repeatedly says: (i) only intersections of world lines are invariant under arbitrary, continuous coordinate transformations, the group of transformations under which the field equations themselves are also to be invariant; (ii) the observations by means of which we test the predictions of any physical theory consist, in principle, of just such coincidences.

When Einstein came back to generally covariant field equations with the tensor equation (13), what is now called the Einstein tensor, he realized that the weak field equation resulting from this tensor involves a metric with more than one variable component. Therefore, such a weak field equation cannot simply be reduced to the classical Poisson equation for one scalar potential, in contradiction to his first expectation. Nevertheless, the equation of motion for a test particle, the geodesic equation reduces in fact, under the mathematical conditions that correspond to Newtonian physics (weak static fields and low velocities), to the Newtonian equation of motion. Under these conditions only one component of the metric tensor, the component g_{44}, enters the equations of motion in first approximation. In this way Einstein was able to surpass the last stumbling block in the fulfillment of his program. On top of that, the fact that exist other variable components of the metric (although they do not affect the equation of motion in the Newtonian limit) is quite significant since it is their existence that explains the perihelion shift of Mercury. After all, the Einstein tensor was compatible with his Correspondence Principle, and the metric associated with the Newtonian limit also explained the perihelion advance of Mercury. What else would Einstein wish for?

> On the basis of the general theory of relativity ... space as opposed to 'what fills space' ... has no separate existence ... There is no such thing as an empty space, i.e., a space without [a gravitational] field ... Space-time does not claim existence on its own, but only as a structural quality of the field. [16]

These words were written late in Einstein's life, and they synthesize his answer to a question that has its origin in 1913, when Einstein was searching for a field equation for gravity. Summarizing, in a schematic mode all we have said before, one may say that Einstein was aware of the possibility of generally covariant field equations, but he believed — wrongly, it turned out — that they could not possess the correct Newtonian limit. He then proposed a field equation covariant under linear coordinate transformations. To sustain his case against generally covariant field equations, Einstein conceived his "hole argument", which alleged to show that a satisfactory generally covariant field equation couldn't exist. Eventually, Einstein came back to generally covariant equations for the gravitational field, and found a way around his hole argument, late in 1915, through the point-coincidence argument, which led him to prove that (classical) gravitational physics is deterministic, although the same physical world can be described by different solutions of the equation of motion. From Einstein discussions and arguments, one may conclude that in General Relativity, general covariance is compatible with determinism only assuming that individual space-time events have no physical meaning by themselves; even the localization on the space-time manifold has no physical meaning, since the points of a manifold are differentiated only by the properties that they inherit from the metric field, i.e., a solution of the generally covariant field equations. Einstein's step toward a profoundly novel understanding of nature is accomplished through his arguments, which can be translated in a very short sentence: no metric, no space-time. Background space-time is eradicated from this new understanding of the world.

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