Dust-filled axially symmetric universes with a cosmological constant

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Following the recent recognition of a positive value for the vacuum energy density and the realization that a simple Kantowski-Sachs model might fit classical tests of cosmology, we study the qualitative behavior of three anisotropic and homogeneous models, Kantowski-Sachs, Bianchi type-I and Bianchi type-III universes, with dust and a cosmological constant, in order to find out which are physically permitted. We find that these models undergo isotropization up to the point that the observations will not be able to distinguish between them and the standard model, except for the Kantowski-Sachs model ($\Omega_{k_0}<0$) and for the Bianchi type-III model ($\Omega_{k_0}>0$) with Ω_{Λ_0} smaller than some critical value Ω_{Λ_M}. Even if one imposes that the Universe should be nearly isotropic since the last scattering epoch ($z\approx 1000$), meaning that the Universe should have approximately the same Hubble parameter in all directions (considering the COBE 4-year data), there is still a large range for the matter density parameter compatible with Kantowski-Sachs and Bianchi type-III models if $|\Omega_0+\Omega_{\Lambda_0}-1|\leq\delta$, for a very small δ. The Bianchi type-I model becomes exactly isotropic owing to our restrictions and we have $\Omega_0+\Omega_{\Lambda_0}=1$ in this case. Of course, all these models approach locally an exponential expanding state provided the cosmological constant $\Omega_\Lambda>\Omega_{\Lambda_M}$.

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I. INTRODUCTION

Over the last five years, the issue of whether or not there is a nonzero value for the vacuum energy density, or cosmological constant, has been raised quite often. Indeed, the possibility of a nonzero cosmological constant Λ has been entertained several times in the past for theoretical and observational reasons (the early history of Λ as a parameter in general relativity has been reviewed by [1], [2], and [3]). Recent supernova results [4,5] strongly support a positive and possibly quite large cosmological constant. Even taking the Hubble constant to be in the range 60–75 km/s/Mpc it is possible to show [6] that the standard model of flat space with a vanishing cosmological constant is ruled out. In a very nice review [7] it is argued that postulating an Ω_Λ-dominated model seems to solve a lot of problems at once. And again, in a quite recent review on the physics and cosmology of the cosmological constant, it is added that ''recent years have provided the best evidence yet that this elusive quantity does play an important dynamical role in the universe'' [8].

On the other hand, if the classical tests of cosmology are applied to a simple Kantowski-Sachs metric and the results compared with those obtained for the standard model, the observations will not be able to distinguish between these models if the Hubble parameters along the orthogonal directions are assumed to be approximately equal [9]. Indeed, as [10] points out, the number of cosmological solutions which demonstrate exact isotropy well after the big bang origin of the Universe is a small fraction of the set of allowable solutions of cosmological equations. It is therefore prudent to take seriously the possibility that the Universe is expanding anisotropically. Note also that some shear free anisotropic models display a Friedmann-Lamaître-Robertson-Walker-(FLRW)-like behavior, as is shown in [11].

II. GLOBAL BEHAVIOR OF THE $\Lambda\neq 0$ MODELS

Taking all this into consideration, we discuss the behavior of some homogeneous but anisotropic models with axial symmetry, filled with a pressureless perfect fluid (dust) and a non-vanishing cosmological constant. For this, we will restrict our study to the metric forms

$$ds^2=-c^2dt^2+a^2(t)dr^2+b^2(t)[d\theta^2+f_k^2(\theta)d\phi^2], \tag{2.1}$$

with the two scale factors $a(t)$ and $b(t)$; k is the curvature index of the 2-dimensional surface $d\theta^2+f_k^2(\theta)d\phi^2$ and can take the values $+1,0,-1$, implying $f_k(\theta)$ equal to $\sin(\theta)$, θ, $\sinh(\theta)$, respectively, giving the following three different metrics: Kantowski-Sachs, Bianchi type-I, and Bianchi type-III [12,13].

Einstein equations for the metric (2.1), for which the matter content is in the form of a perfect fluid and a cosmological term, Λ, are then as follows [12,13]:

$$2\ \frac{\dot a}{a}\ \frac{\dot b}{b}+\frac{\dot b^2}{b^2}+\frac{kc^2}{b^2}=8\pi G\rho+\Lambda c^2, \tag{2.2}$$

$$2\ \frac{\ddot b}{b}+\frac{\dot b^2}{b^2}+\frac{kc^2}{b^2}=-8\pi G\frac{p}{c}+\Lambda c^2, \tag{2.3}$$

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$$\frac{\ddot{a}}{a}+\frac{\ddot{b}}{b}+\frac{\dot{a}}{a}\frac{\dot{b}}{b}=-8\pi G\frac{p}{c}+\Lambda c^2, \tag{2.4}$$

where ρ is the matter density and p is the (isotropic) pressure of the fluid. Here G is Newton's gravitational constant and c is the speed of light. If we consider a vanishing pressure $(p=0)$, which is compatible with the present conditions for the Universe, the last two equations take the form

$$2\frac{\ddot{b}}{b}+\frac{\dot{b}^2}{b^2}+\frac{kc^2}{b^2}=\Lambda c^2, \tag{2.5}$$

$$\frac{\ddot{a}}{a}+\frac{\ddot{b}}{b}+\frac{\dot{a}}{a}\frac{\dot{b}}{b}=\Lambda c^2, \tag{2.6}$$

and Eq. (2.5) can easily be integrated to give

$$\frac{\dot{b}^2}{b^2}=\frac{M_1}{b^3}-\frac{kc^2}{b^2}+\frac{\Lambda}{3}c^2, \tag{2.7}$$

where M_1 is a constant of integration.

The Hubble parameters corresponding to the scale factors $a(t)$ and $b(t)$ are defined by

$$H_a \equiv \dot{a}/a \quad \text{and} \quad H_b \equiv \dot{b}/b.$$

Using them to introduce the following dimensionless parameters, in analogy with what is usually done in FLRW universes, let us define

$$\frac{M_1}{b^3H_b^2}\equiv\Omega_M, \tag{2.8}$$

$$-\frac{kc^2}{b^2H_b^2}\equiv\Omega_k \tag{2.9}$$

and

$$\frac{\Lambda c^2}{3H_b^2}\equiv\Omega_\Lambda. \tag{2.10}$$

The conservation equation (2.7) can now be rewritten as

$$\Omega_M+\Omega_k+\Omega_\Lambda=1. \tag{2.11}$$

Now defining the dimensionless variable $y=b/b_0$ where $b_0=b(t_0)$ is the angular scale factor for the present age of the Universe, and using Eq. (2.11) (taken for $t=t_0$), one may rewrite Eq. (2.7) as

$$\dot{y}=\pm H_{b_0}\sqrt{\Omega_{M_0}\left(\frac{1}{y}-1\right)+\Omega_{\Lambda_0}(y^2-1)+1}, \tag{2.12}$$

where the density parameters, defined previously and H_b with zero subscript, denote as before these quantities at the present time t_0. In this way, a number of independent parameters have been reduced. Substituting Eq. (2.7) into Eq. (2.2) gives

$$\dot{a}=\frac{M_\rho-M_1\dfrac{a}{b}+\dfrac{2}{3}\Lambda c^2ab^2}{2\sqrt{M_1b-kc^2b^2+\dfrac{\Lambda}{3}c^2b^4}}, \tag{2.13}$$

where M_ρ is a constant proportional to matter in the Universe:

$$M_\rho=8\pi G\rho ab^2. \tag{2.14}$$

Using the procedure above, Eq. (2.13) can be rewritten in the form

$$\Omega_\rho-\Omega_M+2\Omega_\Lambda=2\frac{H_a}{H_b}, \tag{2.15}$$

where

$$\Omega_\rho=\frac{M_\rho}{ab^2H_b^2}. \tag{2.16}$$

From Eq. (2.2) one may define a matter density parameter. For this, we introduce the notion of the mean Hubble factor H such that $3H=H_a+2H_b$. Also, for these models, the shear scalar σ [13] is given by

$$\sigma=\frac{1}{\sqrt{3}}(H_a-H_b). \tag{2.17}$$

Thus, Eq. (2.2) may be rewritten [12] as

$$3H^2+\frac{kc^2}{b^2}=8\pi G\rho+\sigma^2+\Lambda c^2. \tag{2.18}$$

As in FLRW universes we call the critical matter density ρ_c when $k=0$ and $\Lambda=0$:

$$\rho_c=\frac{3H^2-\sigma^2}{8\pi G}. \tag{2.19}$$

The matter density is generally defined as $\Omega=\rho/\rho_c$; then,

$$\Omega=\frac{8\pi G\rho}{3H^2-\sigma^2}\equiv\frac{8\pi G\rho}{2H_aH_b+H_b^2}, \tag{2.20}$$

just like in FLRW models, and such that $\Omega=1$ when $k=0$ and $\Lambda=0$, and which is related to Ω_ρ by

$$\Omega=\frac{\Omega_\rho}{1+2\dfrac{H_a}{H_b}}. \tag{2.21}$$

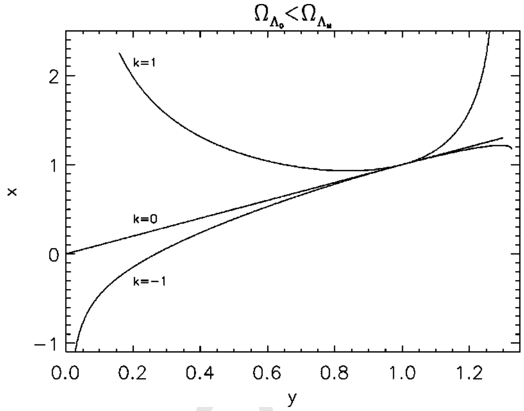

FIG. 1. Scale factors relation, that is, the $x\,y$ dependence for the three models Kantowski-Sachs ($k=1$), Bianchi type-I ($k=0$) and Bianchi type-III ($k=-1$). We show the behavior of $x(y)$ when $\Omega_{\Lambda_0}<\Omega_{\Lambda_M}$. Concretely we have for the Kantowski-Sachs model $\Omega_{M_0}=9$ and $\Omega_{\Lambda_0}=1.5$; for the Bainchi type-I model $\Omega_{M_0}=2$ and $\Omega_{\Lambda_0}=-1$; for the Bainchi type-III model $\Omega_{M_0}=1$ and $\Omega_{\Lambda_0}=-1$.

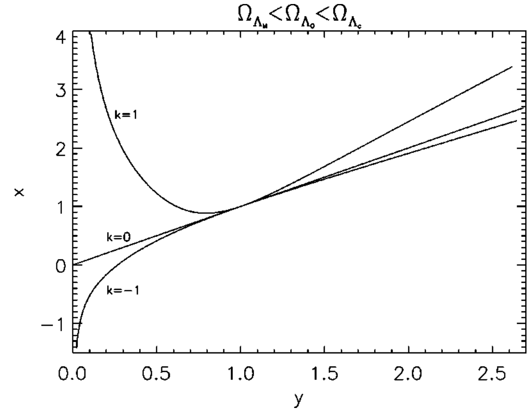

FIG. 2. Scale factors relation, that is, the $x\,y$ dependence for the three models Kantowski-Sachs ($k=1$), Bianchi type-I ($k=0$) and Bianchi type-III ($k=-1$). We show the behavior of $x(y)$ when $\Omega_{\Lambda_M}<\Omega_{\Lambda_0}<\Omega_{\Lambda_c}$. The particular values for the plotting are for the Kantowski-Sachs model $\Omega_{M_0}=2$ and $\Omega_{\Lambda_0}=1.5$; for the Bainchi type-I model $\Omega_{M_0}=0.5$ and $\Omega_{\Lambda_0}=0.5$; for the Bianch type-III model $\Omega_{M_0}=0.2$ and $\Omega_{k_0}=0.4$.

Although Ω_M is not the matter density parameter, it performs the same important role. We emphasize the fact that if for one particular time $H_a=H_b$ and $\Omega_\Lambda=1$, then, by Eqs. (2.11), (2.15), and (2.21), $3\Omega=\Omega_\rho=\Omega_M=-\Omega_k$, and if $0<\Omega_\Lambda\ll 1$ and $\Omega_M=1$, then $-\Omega_k=\Omega_\Lambda$ and $\Omega\simeq 1$.

Introducing another dimensionless variable $x=a/a_0$, Eq. (2.13) takes the form

$$\dot{x}=H_{b_0}\,\frac{\dfrac{\Omega_{M_0}}{2}\left(1-\dfrac{x}{y}\right)+\Omega_{\Lambda_0}(-1+xy^2)+\dfrac{H_{a_0}}{H_{b_0}}}{y\sqrt{\Omega_{M_0}\left(\dfrac{1}{y}-1\right)+\Omega_{\Lambda_0}(y^2-1)+1}}, \tag{2.22}$$

and its number of independent parameters was also reduced, now at the expense of Eq. (2.15) taken for the present time $t=t_0$.

Now, we want to find the time dependence of $b(t)$ in a qualitative way, starting from Eq. (2.12). Since the model universe will be defined only where $\dot{y}^2\geqslant 0$, as was previously done by [14] for FLRW models, the problem is reduced to finding the zeros of $\dot{y}$, with $y\neq 0$.

There are two Ω_Λ values which characterize two zones of distinct behavior for the scale factor b. Starting with condition $\dot{y}=0$ one may obtain

$$\Omega_{\Lambda_0}=\frac{(\Omega_{M_0}-1)y-\Omega_{M_0}}{y^3-y}. \tag{2.23}$$

If we consider $\Omega_{\Lambda_0}=\Omega_{\Lambda_0}(y)$ as a function of y, then this function presents a relative minimum and a maximum, which we will denote by Ω_{Λ_c} and Ω_{Λ_M}, respectively. The relative minimum depends on Ω_{M_0} in the following way [14]: For $\Omega_{M_0}<1/2$ we have

$$\Omega_{\Lambda_c}=\frac{3\Omega_{M_0}}{2}\left\{\left[\sqrt{\frac{(\Omega_{M_0}-1)^2}{\Omega_{M_0}^2}-1}+\frac{1-\Omega_{M_0}}{\Omega_{M_0}}\right]^{1/3}+\frac{1}{\left[\sqrt{(\Omega_{M_0}-1)^2/\Omega_{M_0}^2-1}+(1-\Omega_{M_0})/\Omega_{M_0}\right]^{1/3}}\right\}-(\Omega_{M_0}-1), \tag{2.24}$$

and for $\Omega_{M_0}\geqslant 1/2$ the expression is

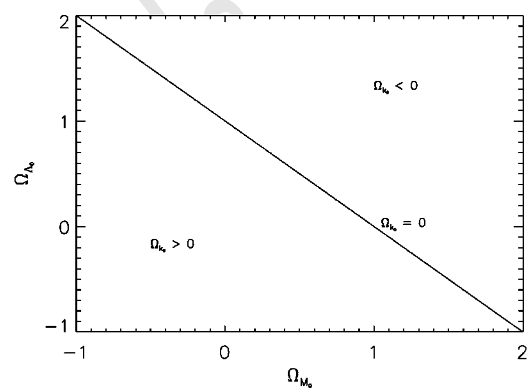

FIG. 3. The Kantowski-Sachs model corresponds to the region above the straight line ($\Omega_{k_0}<0$), the Bianchi type-III model corresponds the region below the straight line; ($\Omega_{k_0}>0$), and the straight line represents the set of region for the Bianchi type-I model ($\Omega_{k_0}=0$).

TABLE I. Density parameters and relative difference between H_a and H_b for Kantowsky-Sachs and Bianchi type-III (B III) models.

	Ω_{M_0}	Ω_{Λ_0}	$\Omega_0+\Omega_{\Lambda_0}$	Ω_{k_0}	$\Delta H/H_a$
KS	1	$\lesssim 1.7\times 10^{-8}$	$1+5.6\times 10^{-9}$	-1.7×10^{-8}	-1.6×10^{-6}
KS	$\lesssim 2\times 10^{-15}$	1	$1+6.7\times 10^{-16}$	-2.0×10^{-15}	-1.4×10^{-6}
KS	$\lesssim 0.3+7.0\times 10^{-9}$	0.7	$1+2.3\times 10^{-9}$	-7.0×10^{-9}	-1.7×10^{-6}
KS	0.3	$\lesssim 0.6+7.0\times 10^{-9}$	$1+2.3\times 10^{-9}$	-7.0×10^{-9}	-1.7×10^{-6}
B III	$1-10^{-10}$	$\gtrsim 9.9\times 10^{-11}$	$1-3.3\times 10^{-13}$	$+1.0\times 10^{-12}$	$+1.3\times 10^{-6}$
B III	$\gtrsim 9.8\times 10^{-14}$	$1-10^{-13}$	$1-6.7\times 10^{-16}$	$+2.0\times 10^{-15}$	$+1.3\times 10^{-6}$
B III	$\gtrsim 0.3-10^{-11}$	0.7	$1-3.3\times 10^{-12}$	$+1.0\times 10^{-11}$	$+1.8\times 10^{-6}$
B III	0.3	$\gtrsim 0.7-10^{-11}$	$1-3.3\times 10^{-12}$	$+1.0\times 10^{-11}$	$+1.8\times 10^{-6}$

$$\Omega_{\Lambda_c}=-3\Omega_{M_0}\cos\left(\frac{\theta+2\pi}{3}\right)-(\Omega_{M_0}-1). \qquad (2.25)$$

The relative maximum is done by

$$\Omega_{\Lambda_M}=-3\Omega_{M_0}\cos\left(\frac{\theta+4\pi}{3}\right)-(\Omega_{M_0}-1), \qquad (2.26)$$

where $\theta=\cos^{-1}[(\Omega_{M_0}-1)/\Omega_{M_0}]$. These expressions are limiting zones of the $(\Omega_{\Lambda_0},\Omega_{M_0})$ plane, where $\dot{y}=0$ has three or one solutions (for details see [14]). The Ω_{Λ_M} expression is also defined for $\Omega_{M_0}>1/2$, but it has the meaning of a maximum only for $\Omega_{M_0}>1$. The Ω_{Λ_0} less than or equal to Ω_{Λ_M} imposes a recollapse of the scale factor b, while greater values produce inflexional behaviors for b. The Ω_{Λ_0} values greater than or equal to Ω_{Λ_c} are physically ''forbidden'' because they do not reproduce the present Universe (see [14]). Obviously, $\Omega_{\Lambda_M}<\Omega_{\Lambda_c}$ always.

Although we are considering anisotropic models, Eq. (2.12) for $\dot{y}$ as a function of Ω_{M_0} is mathematically the same as Eq. (2) obtained by [14] for the homogeneous and isotropic FLRW models. From Eqs. (2.12) and (2.22) one obtains the differential equation

$$\frac{dx}{dy}=\frac{\dfrac{\Omega_{M_0}}{2}\left(1-\dfrac{x}{y}\right)+\Omega_{\Lambda_0}(-1+xy^2)+\dfrac{H_{a_0}}{H_{b_0}}}{\Omega_{M_0}(1-y)+\Omega_{\Lambda_0}(y^3-y)+y}. \qquad (2.27)$$

This equation automatically complies with the two conservation equations (2.11) and (2.15) evaluated at t_0. There are some particular values of the parameters $(\Omega_{M_0},\Omega_{\Lambda_0})$ for which this equation has exact solutions. However, for the majority of the values of parameters, the solution has only been obtained by numerical integration.

We may admit that at a certain moment of time, which we can take as the present time t_0, the Hubble parameters along the orthogonal directions may be assumed to be approximately equal, $H_a\simeq H_b$, even though we started with an anisotropic geometry. This hypothesis has been considered in [9] for the case of a Kantowski-Sachs (KS) model. From this study was derived the conclusion that the classical tests of cosmology are not at present sufficiently accurate to distinguish between a FLRW model and the KS model defined in that paper, with $(H_{a_0}\simeq H_{b_0})$, except for small values of b_0.

Taking $H_{a_0}=H_{b_0}$ for simplicity, one can then integrate Eq. (2.27) and find three different solutions, one for each k value. Figures 1 and 2 show the three kinds of behaviors as a result of integration.

The behavior for the Kantowski-Sachs and Bianchi type-III cases depends on the Ω_{Λ_0} value. If $\Omega_{\Lambda_0}\leqslant\Omega_{\Lambda_M}$, there will be a maximum value for $y,(y_m)$, and since then $\dot{y}(y_m)=0$, the slope of the curve $x=x(y)$ will be infinite at that point. Specifically, we have $\dot{x}(y_m)=+\infty$ for the Kantowski-Sachs model and $\dot{x}(y_m)=-\infty$ for the Bianchi type-III model, even though $x(y_m)$ is finite. When $\Omega_{\Lambda_M}<\Omega_{\Lambda_0}<\Omega_{\Lambda_c}$, after $x=y=1$ is reached we find an almost linear relation between the two scale factors x and y for the two models, while for the Bianchi type-I model we have $x=y$ for the present restrictions. So we see that for the KS model, the scale factor $a(t)$ starts from infinity if $b(t)$ starts from zero. For the Bianchi type-I model, the scale factors are always proportional or even equal. In this situation we do not have an anisotropic model; in fact, we can easily prove that this model is isotropic by a proper reparametrization of the coordinates. For the Bianchi type-III model, the scale factor $b(t)$ never starts from zero, but has an initial value different from zero when a

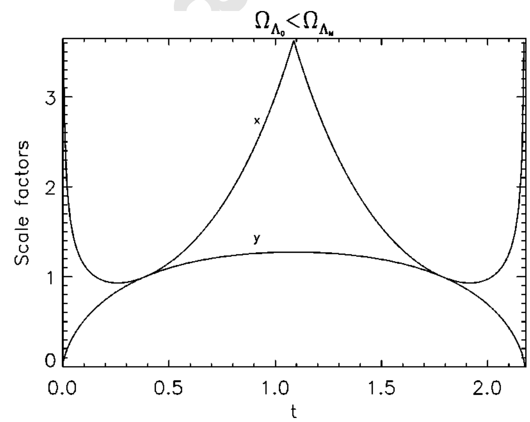

FIG. 4. The scale factors x and y for the Kantowski-Sachs model $(\Omega_{k_0}<0)$ when $\Omega_{\Lambda_0}<\Omega_{\Lambda_M}$. For the plotting we put $\Omega_{M_0}=9$ and $\Omega_{\Lambda_0}=1.5$.

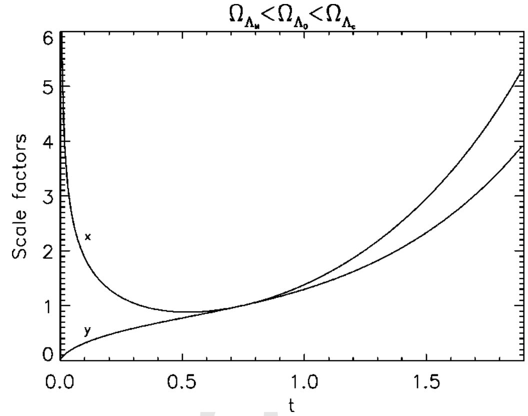

FIG. 5. The scale factors *x* and *y* for the Kantowski-Sachs model ($\Omega_{k_0}<0$) when $\Omega_{\Lambda_M}<\Omega_{\Lambda_0}<\Omega_{\Lambda_c}$. For the plotting we put $\Omega_{M_0}=2$ and $\Omega_{\Lambda_0}=2$.

is null. The following plot shows the zones in 2-dimensional parameter space (Ω_{M_0},Ω_{Λ_0}) where each model is allowed (Fig. 3).

Taking into account the analysis given in [14], we may easily derive the qualitative behavior of $y(t)$, since our equation (2.12) is mathematically equivalent to his equation (3). Now, going back to Figs. 1 and 2, one can then determine the $x(t)$. The plotting below summarizes the several possibilities for the three models: Kantowski-Sachs, Bianchi type-I and Bianchi type-III models, respectively.

The present technology allows us to ''see'' the epoch of last scattering of radiation at a redshift of about 1000; i.e., we can actually observe the most distant information that the Universe provides. The high level of isotropy observed from the cosmic microwave background radiation (CMBR) [15] from this epoch to our present time imposes that the two Hubble factors H_a and H_b must remain approximately equal from this epoch to the present. In other words, we must impose a high isotropy level from the last scattering onwards, in our expressions, i.e.,

$$\frac{\Delta H}{H_a}\equiv\frac{H_a-H_b}{H_a},$$

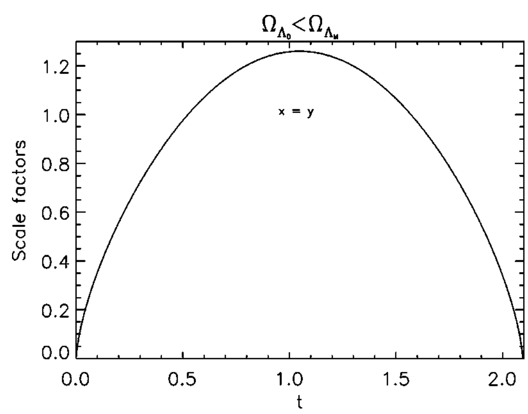

FIG. 6. The scale factors *x* and *y* for the Bianchi type-I model ($\Omega_{k_0}=0$) when $\Omega_{\Lambda_0}<0$. For the plotting we put $\Omega_{M_0}=2$ and $\Omega_{\Lambda_0}=-1$.

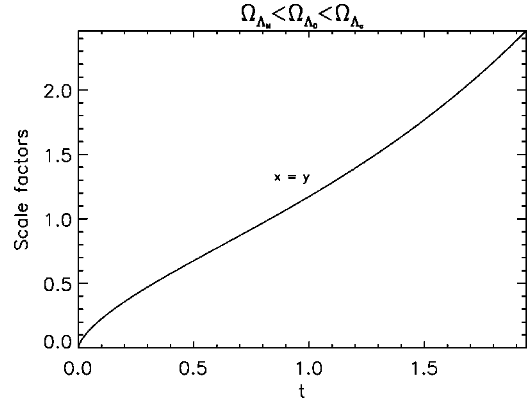

FIG. 7. The scale factors *x* and *y* for Bianchi type-I model ($\Omega_{k_0}=0$) when $\Omega_{\Lambda_0}\geqslant 0$. For the plotting we put $\Omega_{M_0}=0.5$ and $\Omega_{\Lambda_0}=0.5$.

such that $|\Delta H/H_a|\ll 1$. From the Cosmic Background Explorer (COBE) 4-year data [16,17], we have $(\sigma/H)_0\sim 10^{-9}$ and for the last scattering epoch $(\sigma/H)_{ls}\sim 10^{-6}$. At the last scattering we may still consider $H_a\simeq H_b\simeq H$ (H defined above).

We computed several numerical integrations, with Eq. (2.27), in the following way: we gave values to Ω_{M_0} and Ω_{Λ_0} and integrated back in time, from now to the last scattering epoch. These Ω_{M_0} and Ω_{Λ_0} values were chosen such that at the last scattering epoch we had $|\Delta H/H|_{ls}\equiv|1-(dy/dx)_{ls}|=1.7\times 10^{-6}$ or $(dx/dy)_{ls}=1\pm 1.7\times 10^{-6}$. To do this we implemented a 8th order Runge-Kutta method [18].

We concluded that the sum $\Omega_{M_0}+\Omega_{\Lambda_0}$ must be close to unity from above for Kantowski-Sachs model and from below for the Bianchi type-III models.[1] We summarize in Table I the result of imposing $|\Delta H/H|_{ls}\sim 1.7\times 10^{-6}$ for Kantowski-Sachs and Bianchi type-III models, supposing $H_{a_0}=H_{b_0}$ [because $(\sigma/H)_0\sim 10^{-9}$].

From Table I we concluded that all combinations of $\Omega_0+\Omega_{\Lambda_0}$ near unity are equally acceptable for reproducing a small anisotropy $[(\sigma/H)_{ls}\sim 10^{-6}]$ at the last scattering. Nevertheless, we paid special attention to the values of $\Omega_0\sim 0.3$ and $\Omega_{\Lambda_0}\sim 0.7$, since they reproduce a better fit to recent observations [19]. We have in this scenario $|\Delta H/H|<2\times 10^{-6}$ for Kantowski-Sachs and Bianchi type-III universes. All these models approach locally an exponentially expanding state [20] provided the cosmological constant if we consider $\Omega_\Lambda>\Omega_{\Lambda_M}$.

III. CONCLUSIONS

For the Kantowski-Sachs model ($\Omega_{k_0}<0$) (see Figs. 4 and 5), we conclude that if the scale factor $b(t)$ starts from zero, then the scale factor $a(t)$ will start from infinity and

[1]It is obvious that for the Bianchi type-I model ($\Omega_{M_0}+\Omega_{\Lambda_0}=1$), with our restrictions, we have always $\Delta H/H_a=0$.

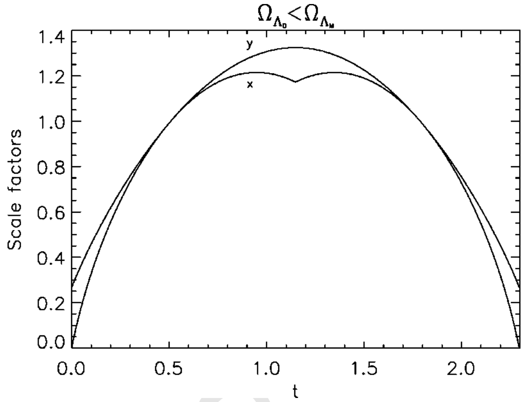

FIG. 8. The scale factors x and y for the Bianchi type-III model ($\Omega_{k_0}>0$) when $\Omega_{\Lambda_0}<\Omega_{\Lambda_M}$. For the plotting we put $\Omega_{M_0}=1$ and $\Omega_{\Lambda_0}=-1$.

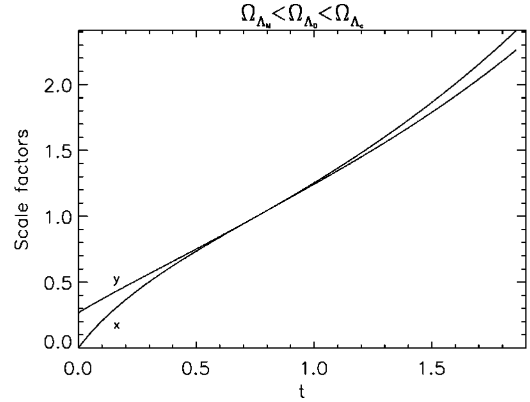

FIG. 9. The scale factors x and y for the Bianchi type-III model ($\Omega_{k_0}>0$) when $\Omega_{\Lambda_M}<\Omega_{\Lambda_0}<\Omega_{\Lambda_c}$. For the plotting we put $\Omega_{M_0}=0.2$ and $\Omega_{\Lambda_0}=0.4$.

decrease afterwards. When $\Omega_{\Lambda_0}<\Omega_{\Lambda_M}$, $b(t)$ reaches the maximum value recollapsing after that. So $a(t)$ will reach a relative maximum when $b(t)$ is maximum (see Fig. 4). After that, when $b(t)=0$, $a(t)$ goes to infinity again. When $\Omega_{\Lambda_M}<\Omega_{\Lambda_0}<\Omega_{\Lambda_c}$, the scale factor $b(t)$ grows indefinitely, giving place to an inflationary scenario. Then, $a(t)$ decreases, reaching a minimum value and growing after that indefinitely, and becoming proportional to $b(t)$ (see Fig. 5). The initial singularity is of a ''cigar'' type.

For the Bianchi type-I model ($\Omega_{k_0}=0$) (see Figs. 6 and 7), the scale factors $a(t)$ and $b(t)$ are proportional or even equal. Thus, this model turns out to be an isotropic one (owing to our restrictions) and $\Omega_0+\Omega_{\Lambda_0}=1$. However, when $\Omega_{\Lambda_0}<\Omega_{\Lambda_M}$, $a(t)$ and $b(t)$ reach the maximum and recollapse after that. And when $\Omega_{\Lambda_M}<\Omega_{\Lambda_0}<\Omega_{\Lambda_c}$, $a(t)$ and $b(t)$ grow indefinitely after an inflection.

For the Bianchi type-III model ($\Omega_{k_0}>0$) (see Figs. 8 and 9), when $\Omega_{\Lambda_0}<\Omega_{\Lambda_M}$, $b(t)$ starts from an initial non vanishing value $[b(t=0)=b_0>0]$, reaching a maximum and recollapsing after that until reaches the same value for $t=0$. Also, $a(t)$ has a similar behavior, but starts from zero and recollapses to zero; nevertheless, $a(t)$ exhibits a relative minimum when $b(t)$ is maximum. When $\Omega_{\Lambda_M}<\Omega_{\Lambda_0}<\Omega_{\Lambda_c}$, $b(t)$ starts again from a non-vanishing value ($b_0>0$), growing indefinitely with an inflection. In this case, $a(t)$ starts from zero and grows indefinitely, becoming approximately proportional to $b(t)$. So the initial singularity is of a ''pancake'' type.

In conclusion, these models undergo isotropization, becoming an asymptotically FLRW universe, except for the Kantowski-Sachs model ($\Omega_{k_0}<0$) with $\Omega_{\Lambda_0}<\Omega_{\Lambda_M}$ and for the Bianchi type-III model ($\Omega_{k_0}>$) with $\Omega_{\Lambda_0}<\Omega_{\Lambda_{M_0}}$. Taking into account the accuracy of the measurements of anisotropy on the one hand and the fact that we can always adjust the density parameters such that $|\Omega_0+\Omega_{\Lambda_0}-1|=\delta$, with $\delta\sim10^{-9}$ on the other, we conclude that these models still stand as good candidates to describe the observed Universe.

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